Statistics

Lecture 9

Feb 19-8:47 AM

SG 14 & 15

Data

1) Qualitative
(Non-Numerical)

2) Quantitative
(Numerical)

1) Discrete
(Countable)

2) Continuous
(Measureable)

Jul 10-4:36 PM

Let x be a discrete random Variable with prob. dist. $P(x)$.

What is Prob. distribution?

It is a way to provide prob. of all possible outcomes from Sample Space.

1) It could be in a form of a chart.

2) It could be in a form of a graph

3) It could be using certain formula

4) we could use def. of prob. as well.

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Some rules:

1) $0 \leq P(x) \leq 1$

2) $P(x) = 1 \iff$ Sure event

3) $P(x) = 0 \iff$ Impossible event

4) $0 < P(x) \leq .05 \iff$ Rare event

5) $\sum P(x) = 1$ Sum of all prob. $= 1$

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Consider the chart below

x	$P(x)$
1	.2
2	.5
3	.3

1) Verify $\sum P(x) = 1$

$.2 + .5 + .3 = 1$ ✓

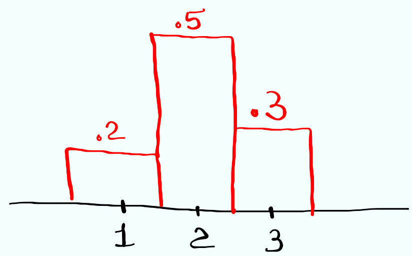
2) $P(x \le 2) = .2 + .5 = \boxed{.7}$

3) $P(x \ge 2) = .5 + .3 = \boxed{.8}$

4) Draw Prob. dist. histogram.

$x \to$ Midpt

$P(x) \to$ Rel. F.



Consider the chart below for x with prob. dist. $P(x)$.

x	$P(x)$
1	.1
2	.2
3	.4
4	.3

1) Find $P(x=4)$

$= 1 - [.1 + .2 + .4]$

Total prob. 1

$= 1 - .7 = \boxed{.3}$

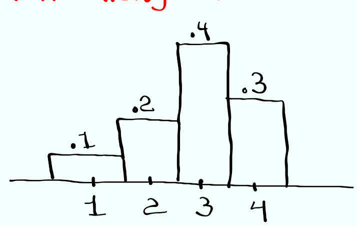
2) $P(x=1 \text{ or } x=4) = .1 + .3 = \boxed{.4}$

3) $P(x < 3) = .1 + .2 = \boxed{.3}$

4) Draw Prob. dist. histogram

$x \to$ Midpt

$P(x) \to$ Rel. F.



Complete the chart below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.3	.3	.3
2	.4	.8	1.6
3	.3	.9	2.7

1) $\sum P(x) = 1$

2) $\sum xP(x) = 2$

3) $\sum x^2 P(x) = 4.6$

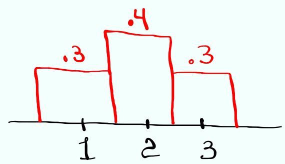
4) Compute $\sum x^2 p(x) - \left(\sum x p(x)\right)^2 = 4.6 - 2^2 = \boxed{.6}$

5) $\sqrt{\text{Last answer}} = \sqrt{.6} \approx \boxed{.775}$

6) Draw Prob. dist. Histogram

$x \to$ Midpt

$P(x) \to$ Rel. F.



Symmetric

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A piggy bank has 4 dimes and 2 nickels.

Randomly take 2 coins, No replacement

Sample Space

NN ND DN DD

Sample space in total value in cents

10¢ 15¢ 20¢

$P(10¢) = P(NN) = \frac{2}{6} \cdot \frac{1}{5} = \boxed{\frac{1}{15}}$

$P(15¢) = P(ND \text{ or } DN) = 2 \cdot \frac{2}{6} \cdot \frac{4}{5} = \boxed{\frac{8}{15}}$

$P(20¢) = P(DD) = \frac{\cancel{4}^{2}}{\cancel{6}_{2}} \cdot \frac{\cancel{3}}{5} = \frac{2 \cdot 3}{5 \cdot 3} = \boxed{\frac{6}{15}}$

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Total ¢	P(Total ¢)
10	$\frac{1}{15}$
15	$\frac{8}{15}$
20	$\frac{6}{15}$

P(10¢ or 20¢) =

$$= \frac{1}{15} + \frac{6}{15} = \boxed{\frac{7}{15}}$$

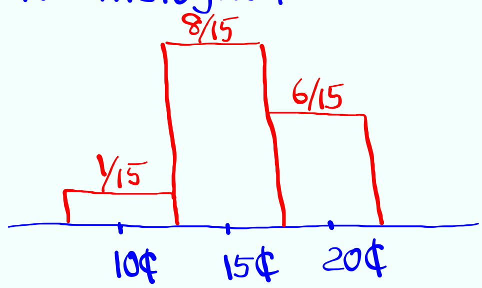
P(get at least 15¢)

$$= \frac{8}{15} + \frac{6}{15} = \boxed{\frac{14}{15}}$$

Draw Prob. dist. Histogram

Total → Mid pt

P(Total) → Rel. F.



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Mean μ "mu"

Variance σ^2 "sigma^2"

Standard deviation σ "sigma"

$$\boxed{\mu = \sum x\,p(x)}$$

$$\boxed{\sigma^2 = \sum x^2 p(x) - \mu^2}$$

$$\boxed{\sigma = \sqrt{\sigma^2}}$$

Jul 10-5:20 PM

Complete the chart below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.3	.3	.3
2	.5	1.0	2.0
3	.2	.6	1.8

1) Verify $\sum P(x) = 1$

$.3 + .5 + .2 = 1 \checkmark$

2) Find μ

$\mu = \sum x\,p(x)$

$= .3 + 1.0 + .6 = \boxed{1.9}$

3) Find σ^2

$\sigma^2 = \sum x^2 p(x) - \mu^2$

$= 4.1 - 1.9^2 = \boxed{.49}$

4) Find σ

$\sigma = \sqrt{\sigma^2}$

$= \sqrt{.49} = \boxed{.7}$

Clear All lists $\boxed{\text{STAT}}\ \boxed{\rightarrow}$ CALC

$x \rightarrow L1$ $\boxed{\text{1: 1-Var Stats}}$

$P(x) \rightarrow L2$ use L1 & L2

$\mu = \bar{x} = 1.9$

$\sigma = \sigma_x = .7$

Total Prob. $\rightarrow$ $n = 1$

$\sigma^2 = .49 = \frac{49}{100}$

what about σ^2?

$\boxed{\text{VARS}}\ \boxed{\text{5: Statistics}}\ \boxed{4{:}\ \sigma_x}\ \boxed{x^2}\ \boxed{\text{Enter}}$

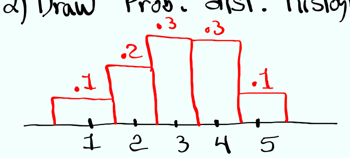
Jul 10-5:23 PM

Given the chart below

x	$P(x)$
1	.1
2	.2
3	.3
4	.3
5	.1

1) Verify $\sum P(x) = 1 \checkmark$

2) Draw Prob. dist. histogram



3) Find μ, σ, and σ^2.

Clear All lists $\boxed{\text{STAT}}\ \boxed{\rightarrow}$ CALC

$x \rightarrow L1$ $\boxed{\text{1: 1-Var Stats}}$

$P(x) \rightarrow L2$ with L1 & L2

$\mu = \bar{x} = 3.1$

$\sigma = \sigma_x = 1.136$

$n = 1$

$\sigma^2 = \frac{129}{100}$

now σ^2

$\boxed{\text{VARS}}\ \boxed{\text{5: Statistics}}\ \boxed{4{:}\ \sigma_x}\ \boxed{x^2}\ \boxed{\text{Enter}}$

$\sigma^2 = 1.29$ in reduced fraction

$\boxed{\text{Math}}\ \boxed{1{:}\blacktriangleright\text{Frac}}\ \boxed{\text{Enter}}$

Jul 10-5:36 PM

3 Females 5 Males Select 2 people
No replacement

Sample Space

FF FM MF MM

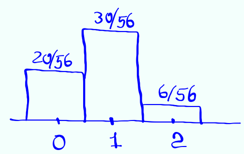
Let x be # of Females Selected,

FF → $x=2$ $P(x=2)=P(FF)=\frac{3}{8}\cdot\frac{2}{7}=\frac{6}{56}$

FM, MF → $x=1$ $P(x=1)=P(FM \text{ or } MF)=\cdots=\frac{30}{56}$

MM → $x=0$ $P(x=0)=P(MM)=\frac{5}{8}\cdot\frac{4}{7}=\frac{20}{56}$

#F, x	P(#F, x)
2	6/56
1	30/56
0	20/56



x → L1
$P(x)$ → L2

$\boxed{\text{1-Var Stats}}$ with L1 & L2

$\mu=\bar{x}=.75$
$\sigma=\sigma_x=.634$
$n=1$

Now find σ^2 in reduced fraction

$\boxed{\text{VARS}}\boxed{\text{5: Statistics}}$
$\boxed{4:\sigma x}\ \boxed{x^2}\boxed{\text{Math}}$
$\boxed{1:\triangleright\text{Frac}}\boxed{\text{Enter}}$

$\sigma^2=\frac{45}{112}$ ✓

Jul 10-5:58 PM

Application

Expected Value → μ → $\bar{x}$

25 Students, 25 tickets were Sold for \$10

one ticket is randomly drawn

The winner gets a Calc. worth \$100.

Net	P(Net)
10 − 100	1/25
10 − 0	24/25

Net → L1
P(Net) → L2

$\boxed{\text{1-VAR Stats}}$
L1, L2

E.V. $=\mu=\bar{x}$
Per Ticket

$\boxed{6}$

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Aidan is going on a trip.

He pays \$100 to insure his luggage.

Any damages, Air line pays him \$1000

P(any damage) = .5%

Net(Airline)	P(Net)	
100 − 1000	.5% = .005	Damage
100 − 0	99.5% = .995	$\overline{\text{Damage}}$

E.V. Per Policy Sold $= \mu = \bar{x}$ $\boxed{\$95}$

Net → L1 $\boxed{\text{1 − Var Stats}}$

P(Net) → L2 L1 & L2

Jul 10-6:17 PM

Pay \$5, draw a Card from a full deck of playing Cards

If you draw an Ace → I give you \$25

If " " a Face → " " " \$5

Any other Card → I give you nothing.

Net (Player)	P(Net)	
25 − 5	4/52	Ace
5 − 5	12/52	Face
0 − 5	36/52	Any other Card

Net → L1 $\boxed{\text{1 − Var Stats}}$ with L1 & L2

P(Net) → L2

SG 14 & SG 15

E.V. Per play For player $= \mu = \bar{x}$ **−\$1.92**

Player expect to lose \$1.92 per play.

Jul 10-6:24 PM

P(any student pass the math class) $= .6$

$P(\text{Pass}) = .6$ $\quad$ $P(\text{Fail}) = .4$

Two students randomly Selected

PP $\quad$ PF $\quad$ FP $\quad$ FF

$P(\text{Both pass}) = P(PP) = (.6)(.6) = \boxed{.36}$

$P(\text{At least one pass}) = 1 - P(\text{No Pass})$

$= 1 - P(FF)$

$= 1 - (.4)(.4)$

$= 1 - .16$

$= \boxed{.84}$

Jul 10-6:32 PM

$P(\text{Math}) = .6$

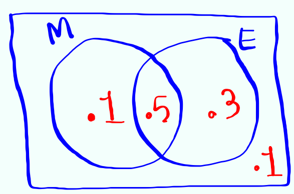
$P(\text{English}) = .8$

$P(\text{Math and English}) = .5$

$.6 - .5 = .1$

$.8 - .5 = .3$

Venn Diagram



Total = 1

$P(\text{Math or English}) = .6 + .8 - .5 = \boxed{.9}$

$P(\text{English} \mid \text{Math}) = \dfrac{P(M \text{ and } E)}{P(M)} = \dfrac{.5}{.6} = \dfrac{5}{6}$

$= \boxed{.833}$

$P(\text{Math} \mid \text{English}) = \dfrac{P(M \text{ and } E)}{P(E)} = \dfrac{.5}{.8} = \dfrac{5}{8}$

$= \boxed{.625}$

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$P(A) = .2$

$P(B) = .4$

$\boxed{P(A|B) = .25}$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$.25 = \frac{P(A \text{ and } B)}{.4}$$

Cross-Multiply

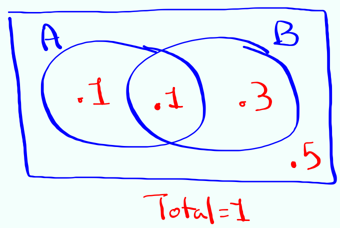
$P(A \text{ or } B) = P(A) + P(B) - \boxed{P(A \text{ and } B)}$

$P(A \text{ and } B) = (.25)(.4)$

$= \boxed{.1}$

$= .2 + .4 - .1$

$= \boxed{.5}$



$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$= \frac{.1}{.2} = \frac{1}{2} = \boxed{.5}$$

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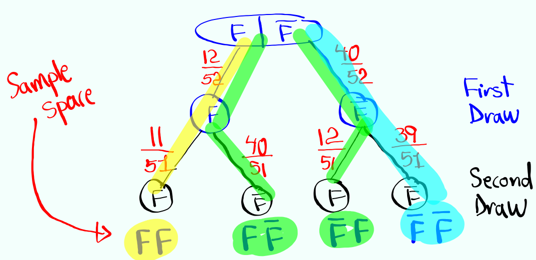
Draw 2 Cards from a full deck of Playing Cards, No replacement.

Make Tree diagram for Selecting Face Card



$$P(FF) = \frac{12}{52} \cdot \frac{11}{51} = \boxed{\frac{11}{221}}$$

$$P(1 \text{ Face } \& \ 1 \ \overline{\text{Face}}) = 2 \cdot \frac{12}{52} \cdot \frac{40}{51} = \boxed{\frac{80}{221}}$$

$$P(\text{No Face Cards}) = \frac{40}{52} \cdot \frac{39}{51} = \frac{\boxed{130}}{221} = \frac{10}{17}$$

$$P(\underbrace{\text{at least}} \ 1 \text{ Face Card}) = 1 - P(\text{None})$$

$$= 1 - \frac{10}{17} = \boxed{\frac{7}{17}}$$

Jul 10-6:51 PM

# Face	P(# Face)
2	11/221
1	80/221
0	130/221

Face → L1

P(# Face) → L2

$\boxed{\text{1-Var Stats}}$

L1 & L2

$\mu = \bar{x} = .462$

$\sigma = \sigma_x = .590$

$n = 1$

find σ^2 in reduced fraction

$$\boxed{\frac{1000}{2873}}$$

Jul 10-7:03 PM